



SHENTON  
COLLEGE

# SHENTON COLLEGE

Examination Semester One 2017  
Question/Answer Booklet

## MATHEMATICS SPECIALIST UNIT 3

### Section One (Calculator-free)

Your name \_\_\_\_\_

#### Time allowed for this section

Reading time before commencing work: 5 minutes

Working time for paper: 50 minutes

#### Material required/recommended for this section

##### To be provided by the supervisor

Question/answer booklet for Section One.  
Formula sheet.

##### To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters  
Special items: nil

#### Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

#### Structure of this examination

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	11	11	100	99	65
<b>Total</b>				152	100

## Instructions to candidates

The rules for the conduct of Western Australian external examinations are detailed in the Year 12 *Information Handbook 2017*. Sitting this examination implies that you agree to abide by these rules

Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: if you need to use the space to continue an answer, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

**Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than 2 marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

It is recommended that you **do not use pencil, except in diagrams**.

## STRUCTURE OF THIS PAPER

QUESTION	MARKS AVAILABLE	MARKS AWARDED
1	6	
2	5	
3	8	
4	7	
5	7	
6	6	
7	6	
8	8	
<b>TOTAL</b>	<b>53</b>	

**Section One: Calculator-free****35% (53 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

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**Question 1****(6 marks)**

A sphere has its centre at  $R$ , which has position vector  $2\mathbf{i} - \mathbf{k}$ . If the diameter of the sphere is  $\overrightarrow{PQ}$  and the position vector of  $Q$  is  $8\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ .

(a) Determine the position vector of  $P$ . (2 marks)

(b) Determine the vector equation of the sphere. (2 marks)

(c) The sphere intersects the  $y$ -axis where  $y = a$ . Determine the value(s) of the constant  $a$ . (2 marks)

**Question 2****(5 marks)**

A function is defined by  $g(z) = 2z^4 - z^3 + 7z^2 - 4z - 4$ .

(a) Show that  $z = 1$  and  $z = 2i$  are both zeros of  $g(z)$ . (2 marks)

(b) Determine all solutions to  $g(z) = 0$ . (3 marks)

**Question 3****(8 marks)**Simplify the following into the form  $x + iy$ .

(a)  $\frac{3}{2i} + 2i$ .

**(2 marks)**

(b)  $\frac{1}{(2-i)^2}$ .

**(3 marks)**

(c)  $(-\sqrt{2} + \sqrt{2}i)^6$ .

**(3 marks)**

## Question 4

(7 marks)

The function  $f$  is defined by  $f(x) = \frac{1}{1-x}$ .

(a) Evaluate  $f(f(-1))$ . (1 mark)

(b) Determine and simplify an expression for  $f \circ f(x)$ . (2 marks)

(c) For  $f \circ f(x)$ , state the

(i) domain. (2 marks)

(ii) range. (2 marks)

**Question 5****(7 marks)**

(a) The equation  $2z^2 + 3z + 5 = 0$  has roots of  $\alpha$  and  $\beta$ . Determine the value of

(i)  $\alpha + \beta$ . (1 mark)

(ii)  $\alpha\beta$ . (1 mark)

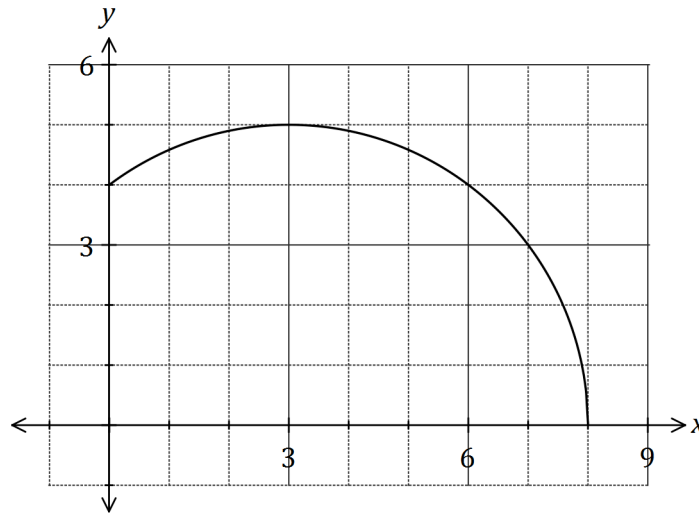
(iii)  $2\alpha^2 + 3\alpha + 5$ . (1 mark)

(b) Determine the values of the real constants  $a$  and  $b$  if  $z - 2 + i$  is a factor of  $z^3 + az + b$ . (4 marks)

## Question 6

(6 marks)

Let  $f(x) = \sqrt{16 + 6x - x^2}$ ,  $0 \leq x \leq 8$ . The graph of  $y = f(x)$  is shown below.



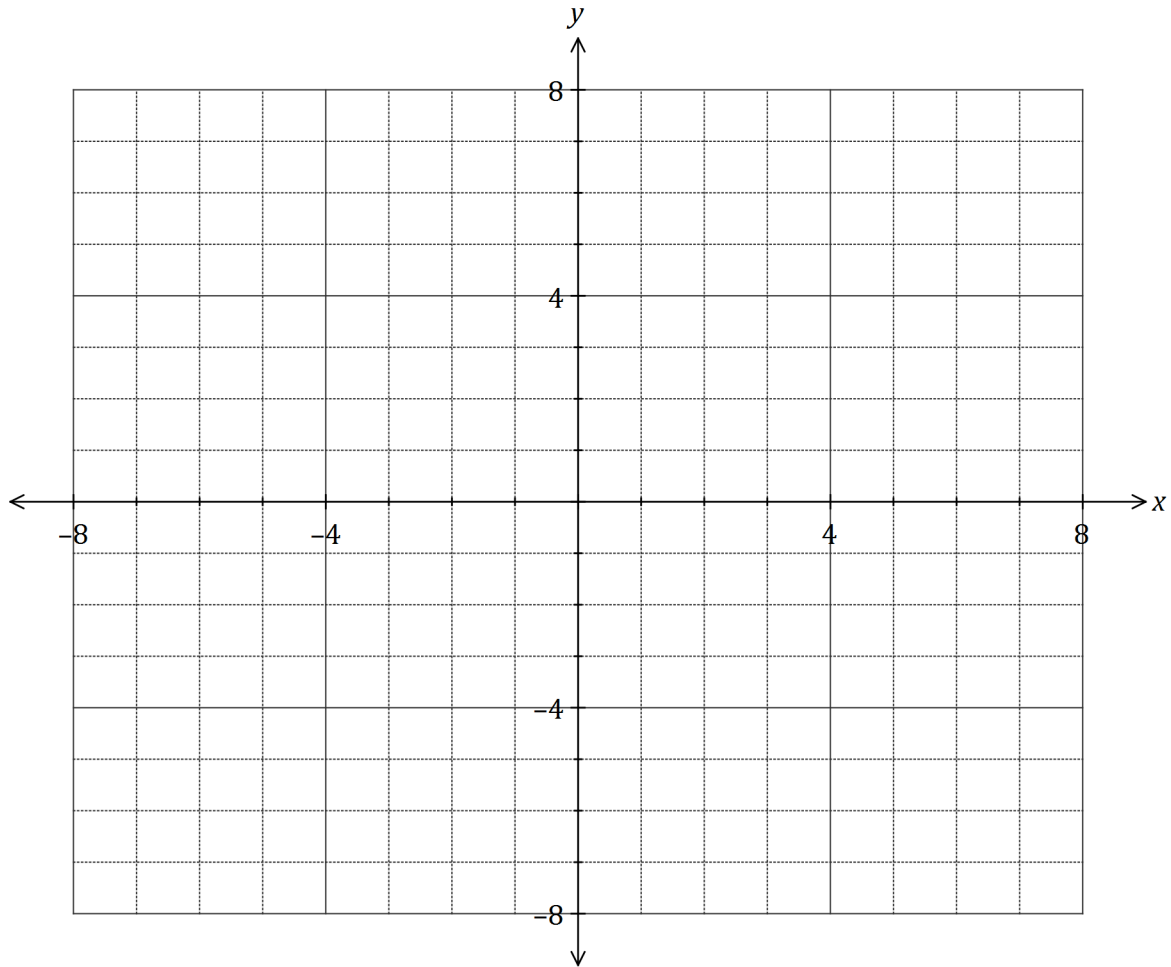
- (a) In order that  $y = f^{-1}(x)$  is a function, the domain of  $f$  must be restricted to  $k \leq x \leq 8$ . Explain why this restriction is necessary and state the minimum value of  $k$ . (2 marks)
- (b) Using the restriction from (a), determine the inverse function of  $f$  and its domain. (4 marks)



Question 7

(6 marks)

On the axes below, draw the graph of  $y = \frac{x^2}{x^2 - 2x - 3}$ , clearly showing key features and the behaviour of the curve near the asymptotes.



## Question 8

(8 marks)

- (a) Two of the solutions to the equation  $z^n = 1, n \in \mathbb{Z}^+$ , are  $z = \text{cis} \frac{\pi}{2}$  and  $z = \text{cis} \frac{\pi}{3}$ .
- (i) State another solution to the equation. (1 mark)
- (ii) Determine, with reasons, the minimum value of  $n$ . (2 marks)
- (ii) If  $z = \text{cis} \frac{11\pi}{24}$  is not a solution to the equation then determine, with reasons, the maximum value of  $n$ . (2 marks)
- (b) If  $z = \text{cis} \frac{\pi}{4}$ , determine the sum of the geometric series  $1 + z + z^2 + z^3 + \dots + z^{24}$ . Explain your answer. (3 marks)

Additional working space

Question number: \_\_\_\_\_

